• Textbooks, cell phones, or any other forms of wireless communication are strictly prohibited in this or any exam. Giving or receiving aid in an examination is cause for dismissal from the University. Any other violation of academic honesty can have the same effect.

• Answer all multiple-choice questions on the sheet provided. You can use this exam booklet as scratch paper, but your written work for the multiple choice questions will not be graded.

• For the free-response question, perform the necessary calculations in the space provided. If additional space is needed, use the back of the question sheets.

• ALL WORK MUST BE SHOWN FOR THE FREE RESPONSE QUESTION IN ORDER TO RECEIVE FULL CREDIT.

• Numerical answers must include appropriate units.

Advice: Read each a question completely before doing any part of it.

Total of 11 pages (including 1st page)

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1. When a hard rubber rod is given a negative charge by rubbing it with wool:
   a. positive charges are transferred from rod to wool
   b. negative charges are transferred from rod to wool
   c. positive charges are transferred from wool to rod
   d. negative charges are transferred from wool to rod
   e. negative charges are created and stored on the rod
   \textbf{Answer : d}

2. Charges \( Q, \ -Q, \) and \( q \) are placed at the vertices of an equilateral triangles as shown. The total force exerted on the charge \( q \) is:
   a. toward charge \( Q \)
   b. toward charge \(-Q\)
   c. away from charge \( Q \)
   d. at right angles to the line joining \( Q \) and \(-Q\)
   e. parallel to the line joining \( Q \) and \(-Q\)
   \textbf{Answer : e}

3. A point charge is placed on an electric field which varies with location. No force is exerted on this charge:
   a. at locations where the electric field is zero
   b. at locations where the electric field strength is \( 1/(1.6 \times 10^{-19}) \) N/C
   c. if the charge is moving along a field line
   d. if the charge is moving perpendicular to a field line.
   e. If the field is caused by an equal amount of positive and negative charge
   \textbf{Answer : a}

4. The diagram shows the electric field lines in a region of space containing two small charged spheres (Y and Z). Then:
   a. Y is negative and Z is positive
   b. the magnitude of electric field is the same everywhere
   c. the field is strongest midway between X and Y
   d. a small negatively charged body placed at X would be pushed to the right
   e. Y and Z must have the same sign
   \textbf{Answer : d}

5. Two point charges, \( 8 \times 10^{-9} \) C and \(-2 \times 10^{-9} \) C are separated by 4 m. The electric field intensity (in N/C) midway between them is:
   a. \( 9 \times 10^9 \)
   b. 13,500
   c. 135,000
   d. \( 36 \times 10^9 \)
   e. 22.5
   \textbf{Answer : e}
6. Two point charges are arranged as shown. Where could a third charge +1 C be placed so that the net electrostatic force on it is zero?
   a. I only       b. I and II only     c. III only    d. I and III only    e. II only

Answer: c

7. Choose the INCORRECT statement:
   a. Gauss’s law can be derived from Coulomb’s law
   b. Gauss’s law states that the net number of lines crossing any closed surface in an outward direction is proportional to the net charge enclosed within the surface
   c. Coulomb’s law can be derived from Gauss’s law and symmetry
   d. Gauss’s law applies to a closed surface of any shape
   e. according to Gauss’s law, if a closed surface encloses no charge, then the electric field must vanish everywhere on the Gaussian surface

Answer: e

8. A solid insulating sphere of radius R contains a uniform volume distribution of positive charge. Which of the graph below correctly gives E as a function of r?

Answer: c

9. A wastepaper basket with a 0.15 m radius opening is in a uniform electric field of 300 N/C, perpendicular to the opening. The total flux through the sides and bottom, in N·m²/C, is:
   a. 0       b. 4.2       c. 21       d. 280       e. can’t tell without knowing the areas of the sides and bottom

Answer: c

10. A physics instructor in an anteroom charges an electrostatic generator to 25 µC, then carries it into the lecture hall wall. The net electric flux in N·m²/C through the lecture hall walls is:
    a. 0       b. 25 x 10⁻⁶       c. 2.2 x 10⁵       d. 2.8 x 10⁶       e. can’t tell unless the lecture hall dimensions are given

Answer: d
11. A particle (mass m, charge \(-q\)) is projected with speed \(v_0\) into the region between two parallel plates as shown. The potential difference between the two plates is \(V\) and their separation is \(d\). The change in kinetic energy of the particle as it traverses this region is:
   a. \(-qV/d\)  
   b. \(2qV/mv_0^2\)  
   c. \(qV\)  
   d. \(mv_0^2/2\)  
   e. none of these

   Answer: c

12. During a lightning discharge, 30 C of charge move through a potential difference of \(1.0 \times 10^8\) V in \(2.0 \times 10^{-2}\) s. The energy released by this lightning bolt is:
   a. \(1.5 \times 10^{11}\) J  
   b. \(3.0 \times 10^9\) J  
   c. \(6.0 \times 10^7\) J  
   d. \(3.3 \times 10^6\) J  
   e. 1500 J

   Answer: b

13. Choose the CORRECT statement:
   a. a proton tends to go from a region of low potential to a region of high potential  
   b. the potential of a negatively charged conductor must be negative  
   c. if \(E = 0\) at a point \(P\) then \(V\) must be zero at \(P\)  
   d. if \(V = 0\) at a point \(P\) then \(E\) must be zero at \(P\)  
   e. none of the above are correct

   Answer: e

14. A hollow metal sphere is charged to a potential \(V\). The potential at its center is:
   a. \(V\)  
   b. 0  
   c. \(-V\)  
   d. \(2V\)  
   e. \(\pi V\)

   Answer: a

15. Points R and T are at a distance \(d\) from each of two equal and opposite charges as shown. If \(k = 1/4\pi \varepsilon_0\), the work required to move a negative test charge \(q\) from R to T is:
   a. zero  
   b. \(kqQ/d^2\)  
   c. \(kqQ/d\)  
   d. \(kqQ/(\sqrt{2}d)\)  
   e. \(kqQ/(2d)\)

   Answer: a
16. A parallel plate capacitor is charged and then disconnected from the source. Using insulating handles, the plate are brought closer together. Therefore:
   a. the charge on each plate increased
   b. the charge on each plate decreased
   c. the potential difference between the plates increased
   d. the potential difference between the plates decreased
   e. the capacitance decreased

**Answer:** d

17. Suppose one has available:
   - two sheet of copper
   - a sheet of mica (0.1 mm, \( \kappa = 6 \))
   - a sheet of glass (2 mm, \( \kappa = 7 \))
   - a slab of paraffin (1 cm, \( \kappa = 2 \))

   To obtain the largest capacitance, place between the two copper sheets:
   a. a 1 mm gap of air
   b. the mica
   c. the glass
   d. the paraffin
   e. the mica, glass and paraffin

**Answer:** b

18. A certain wire has resistance \( R \). Another wire, of the same material, has half the length and half the diameter of the first wire. The resistance of the second wire is:
   a. \( R/4 \)
   b. \( R/2 \)
   c. \( R \)
   d. \( 2R \)
   e. \( 4R \)

**Answer:** d

19. Copper conductor which would have the least resistance would be:
   a. thin, long and hot
   b. thick, short and cool
   c. thick long and hot
   d. thin, short and cool
   e. thin, short and hot

**Answer:** b

20. Which of the following graphs best represents the current-voltage relationship of an incandescent light bulb?

   ![Graphs A, B, C, D, E]

**Answer:** a
21. The equivalent resistance between points A and B of the circuit shown is
   a. 4 Ω   b. 4.5 Ω   c. 6 Ω   d. 3 Ω   e. 2.5 Ω

Answer : e

22. The current in the 5.0 Ω resistor in the circuit shown is :
   a. 0.42 A   b. 0.67 A   c. 1.5 A   d. 2.4 A   e. 3.0 A

Answer : c

23. In the diagram below, all bulbs are identical and all cells are identical. In which circuit (A, B, C, D, E) will the bulbs glow with the same brightness as in X?

Answer : d

24. “The sum of the currents flowing into a junction equals the sum of the currents flowing out of the junction” expresses:
   a. Newton’s third law   b. Ohm’s law
   c. Newton’s second law   d. conservation of energy
   e. conservation of charge

Answer : e
25. In the diagram below, all bulbs are identical and all cells are identical. In which circuit will the bulbs be dimmest?

Answer : d

26. The diagram shows six $6 \, \mu F$ capacitors. The capacitance between “a” and “b” is:
   a. $3 \, \mu F$   \quad b. $4 \, \mu F$   \quad c. $6 \, \mu F$   \quad d. $9 \, \mu F$   \quad e. $1 \, \mu F$

Answer : b

27. Let $Q$ denote charge, $V$ denote potential difference and $U$ denote stored energy. Of these quantities, capacitors in parallel must the same:
   a. $Q$ only   \quad b. $V$ only   \quad c. $U$ only   \quad d. $Q$ and $U$ only   \quad e. $V$ and $U$ only

Answer : b

28. In the circuit shown, both resistors have the same value $R$. Suppose switch $S$ is closed. When it is then opened, the circuit has a time constant $\tau_a$. Conversely, suppose $S$ is open. When it is then closed, the circuit has a time constant $\tau_b$. The ratio $\tau_a/\tau_b$ is:
   a. 1   \quad b. 2   \quad c. 0.5   \quad d. 0.667   \quad e. 1.5

Answer : b
29. In the circuit shown, the capacitor is initially uncharged. At \( t = 0 \), switch S is closed. If \( \tau \) denotes the time constant, what is the approximate current through the 3 \( \Omega \) resistor when \( t = \tau/100 \)?
   a. \( \frac{3}{8} \) A  
   b. \( \frac{1}{2} \) A  
   c. \( \frac{3}{4} \) A  
   d. 1 A  
   e. \( \frac{3}{2} \) A

   **Answer : d**

30. An initially uncharged capacitor C is connected in series with resistor R. This combination is then connected to a battery of emf \( V_0 \). Sufficient time elapses so that a steady state is reached. Which of the following statements is NOT true:
   a. the time constant is independent of \( V_0 \)
   b. the final charge on C is independent of R
   c. the total energy dissipated by R is independent of R
   d. the total energy dissipated by R is independent of \( V_0 \)
   e. the initial current (just after battery was connected) is independent of C

   **Answer : c**
Free-response questions

1. (16 points)

Four point charges $+q$, $+q$, $-q$, and $-q$ are replaced at the 4 corners of a square, 10 cm each side. Diagonal corners of the square are occupied by opposite charges. ($q = 5 \text{ nC}$).

(a) Determine the electric field, both magnitude and direction, at the center of the square.

(Solution)

$$E_x = 0 \text{ by symmetry, } E_y = +4 \frac{kq}{a^2} \sin 45^\circ, \text{ where } a \text{ is the side length of the square, i.e. 10 cm.}$$

$$E = 4\sqrt{2} \frac{kq}{a^2} \hat{j} = 4 \times 1.14 \times 9 \times 10^9 \times 5 \times 10^{-9} / 0.1^2 \hat{j} = (2.54 \times 10^4 \text{ N/C}) \hat{j}$$

(b) Determine the electric potential at center of the square.

(Solution) $V(\text{center}) = 0$, because of cancellation

(c) Determine the energy required to assemble these four point charges from far apart to the present configuration.

(Solution)

$$W = \frac{kq^2}{a} \left( +2 - 2 - \frac{2}{\sqrt{2}} \right) = \frac{\sqrt{2}kq^2}{a} = -1.41 \times 9 \times 10^9 \times (5 \times 10^{-9})^2 / 0.01 = -3.18 \times 10^{-5} \text{ J.}$$

(d) An electron mass $m_e = 9.11 \times 10^{-31} \text{ kg}$ is placed at the center of the square with zero initial velocity. Determine its instantaneous acceleration, both direction and magnitude, due to the four point charges at the corners.

(Solution) $\ddot{a} = -eE_y / m_e \hat{j} = -\left( 1.6 \times 10^{-19} \times 2.54 \times 10^4 / 9.11 \times 10^{-31} \right) \hat{j} = (4.47 \times 10^{15} \text{ m/s}^2) \hat{j}$
Three capacitors are connected as shown in the figure. Their capacitances are $C_1 = 10 \ \mu F$, $C_2 = 5 \ \mu F$, and $C_3 = 4 \ \mu F$. The voltage $V_{ab}$ is kept at 100 V by connecting to a battery.

(a) Determine the equivalent capacitance of the combination. (4 pts)

(Solution) $C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{20}{9} \ \mu F$

$\therefore C_{eq} = C_2 + C_{23} = 12.22 \ \mu F$

(b) What is the charge on each capacitor? (4 pts)

(Solution) $Q_1 = C_1 V_{ab} = 10 \ \mu F \times 100 \ V = 1000 \ \mu C$

$Q_2 = Q_3 = C_{23} V_{ab} = \frac{20}{9} \ \mu F \times 100 \ V = \frac{2000}{9} \ \mu C$

(c) What is the potential difference across each capacitor. (6 pts)

$V_1 = V_{ab} = 100 \ V$.

(Solution) $V_2 = \frac{Q_2}{C_2} = \frac{2000}{9} \times \frac{1}{5} = 400/9 \ V$

$V_3 = \frac{Q_3}{C_3} = \frac{2000}{9} \times \frac{1}{4} = 500/9 \ V$

(d) What is the energy stored on each capacitor? (6 pts)

$U_1 = \frac{1}{2} Q_1 V_1 = \frac{1}{2} \times 10^{-3} \times 100 = 0.05 \ J$

(Solution) $U_2 = \frac{1}{2} Q_2 V_2 = \frac{1}{2} \times \left(\frac{2000}{9}\right) \times 10^{-6} \times (400/9) = \frac{4}{810} = 0.0049 \ J$

$U_3 = \frac{1}{2} Q_3 V_3 = \frac{1}{2} \times \left(\frac{2000}{9}\right) \times 10^{-6} \times (500/9) = \frac{5}{810} = 0.0062 \ J$

(e) If each capacitor is filled with a material of dielectric constant $\kappa = 4$, what is the energy stored on this combination of capacitors when $V_{ab}$ is maintained at 100 V. (4 pts)

(Solution) The total $U = U_1 + U_1 + U_1 = 11/180 = 0.0611 \ J$

$U_\kappa = \kappa U = \frac{44}{180} \ J = 0.2444 \ J$
3. (Extra 10 pts)
A metal sphere $A$ with radius $R$ is supported on an insulating stand at the center of a hollow, metal, spherical shell $B$ with inner radius $2R$ and outer radius $3R$. A charge $+q$ resides on the small sphere $A$. On the other hand, the shell $B$ has an overall net charge $-3q$. The space outside $B$, $r>3R$, is free of charge distribution.

(a) Determine the electric field (magnitude and direction) in the following regions of different $r$, (8 pts)

Region I. ($r<R$) : 
(Solution) $\vec{E} = 0$

Region II. ($R<r<2R$) : 
(Solution) $\vec{E} = \left(\frac{kq}{r^2}\right)\hat{r}$

Region III. ($2R<r<3R$) : 
(Solution) $\vec{E} = 0$

Region IV. ($3R<r$) : 
(Solution) $\vec{E} = -2\left(\frac{kq}{r^2}\right)\hat{r}$

(b) Determine the potential $V(R)$ at the surface of the sphere $A$, with $V(\infty)=0$. (2 pts)

(Solution) $V(R) = V(0) = \frac{kq}{R} \left(1 - \frac{1}{2} - \frac{2}{3}\right) = -\frac{kq}{6R}$